Stochastic population modeling

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Fig 4.1, p. 121

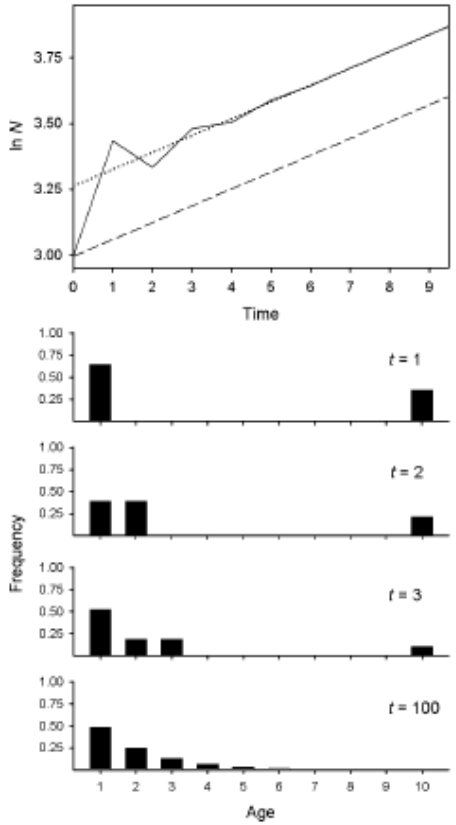

Figure 4.1: Deterministic growth of the total population size (upper panel) and change in age-distribution through time (lower panel) in a deterministic matrix model of the Lefkovitch with 10 stages, corresponding to ages 1-9 and the last stage contains individuals of age 10 and older. Parameter values are $p_i = 0.55$ for all classes, $f_1 = 0$ and $f_i = 1$ for $i > 1$. Initially $n_1 = 20$ and $n_i = 0$ for $i > 1$ at time zero. The dashed line shows the exact exponential growth in the case that the 20 individuals originally were distributed according to the stable age-distribution for the model. The dotted line shows the growth of the reproductive value.

Generation time

Expected age for mothers that give birth when population has a stable age distribution.

$$T = \sum_{i=1}^{k} i \times l_i \times f_i \times \lambda^{-i}$$

4.2.3 Matrix formulation

- **Multiplicative growth rate** of the population is the dominant eigenvalue λ of the projection matrix $\underline{l}$
- **Stable age distribution** $\underline{u}$, $\left(\sum u_i = 1\right)$
  = right dominant eigenvector defined by $\underline{l}\,\underline{u} = \lambda \underline{u}$
- **Reproductive values** $\underline{v}$, $\left(\sum u_i v_i = 1\right)$
  defined by $\underline{v}\,\underline{l} = \lambda \underline{v}$
- **Total reproductive value** after one generation
  $$V + \Delta V = \underline{v}\left(\underline{n} + \Delta \underline{n}\right) = \underline{v}\,\underline{l}\,\underline{n} = \lambda \underline{v}\underline{n} = \lambda V$$

4.2.3 Matrix formulation

Important result from linear algebra: $\frac{\partial \lambda}{\partial l_{i,j}} = v_i u_j$

where $l_{i,j}$ is a non-zero element in $\underline{l}$

This is called the sensitivity of λ with respect to the (i,j)th element

4.3.3 Reproductive value dynamics

$\underline{n}$: population vector

$\underline{v}$: reproductive value defined by

$$\underline{l} = E\underline{M} = EE\left(\underline{M} \mid \underline{Z}\right); \quad \underline{v}\,\underline{l} = \lambda \underline{v}$$

$\underline{u}$: age structure, $\sum u_i = 1; \quad \underline{l}\,\underline{u} = \lambda \underline{u}$

Scaling: $\sum u_i v_i = 1$

$V = \sum n_i v_i$: total reproductive value